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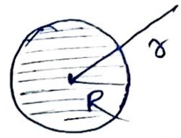
21. Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Compute the gradient of V in each region and check that it yields the correct field. Sketch $V(r)$

$$E_{outside} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau}{r^2} \quad \left(\text{where } r > R\right)$$

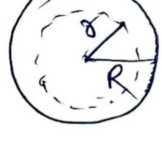


[where ρ = volume charge density

$$E_{outside} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad (r > R)$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3}$$

$$E_{inside} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho\, d\tau}{r^2} \quad \left(\text{where } r < R\right)$$



$$= \frac{1}{4\pi\epsilon_0}\frac{q}{\frac{4}{3}\pi R^3}\left[\frac{4\pi r^3}{3\, r^2}\right]$$

$$E_{inside} = \frac{1}{4\pi\epsilon_0}\frac{q r}{R^3} \quad (r < R)$$

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Potential inside the sphere

$$V(r<R) = -\int_{+\infty}^{r} \vec{E}\cdot d\vec{l} = -\int_{+\infty}^{R} \vec{E}_{outside}\cdot d\vec{l} - \int_{R}^{r} \vec{E}_{inside}\cdot d\vec{l}$$

$$= -\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,dr - \int_{R}^{r}\frac{1}{4\pi\epsilon_0}\frac{q r}{R^3}\,dr$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r}\right]_{\infty}^{R} - \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}\cdot\left[\frac{r^2}{2}\right]_{R}^{r}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} - \frac{1}{\infty}\right) - \frac{q}{4\pi\epsilon_0 R^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right)$$

$$V(r<R) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{r^2 - R^2}{2R^3}\right] \quad (\text{where } r < R)$$

Potential outside the sphere

$$V(R<r) = \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{r}$$

at centre $(r=0)$

$$V(r=0) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{R} + \frac{1}{2R}\right)$$

$$= \frac{3q}{8\pi\epsilon_0 R}$$