A Short Note On

# "Single Slit Fraunhofer Diffraction"

(For 2nd Semester Physics-Hons. Students)

By


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# ***Fraunhofer Diffraction:***

The diffraction where both the source and the screen are at infinite distance is known as Fraunhofer diffraction. Here the convex lens are used to make both the incident and emerging rays parallel. The incident beams are planner here.

## ***Single slit diffraction:***

### ***Intensity distribution function (IDF):***

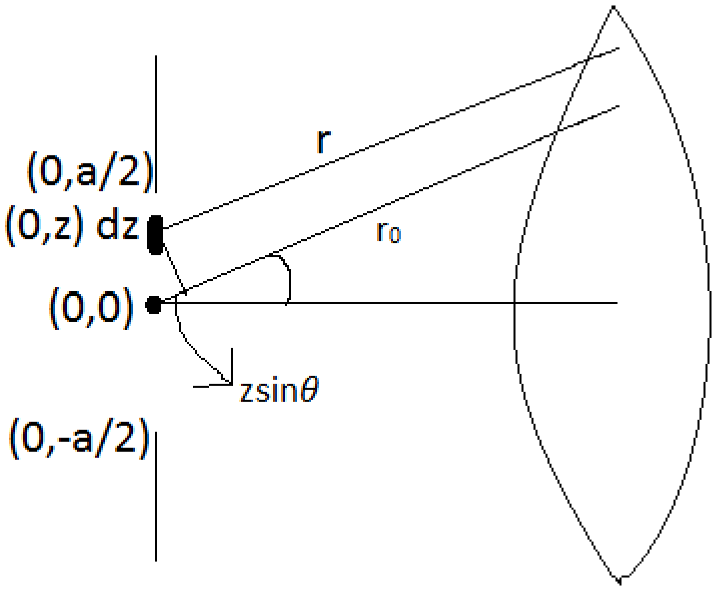
We shall first calculate the intensity distribution function (IDF) for single slit diffraction.

The amplitude of vibration of the secondary wavelets on the slit is given by

$$y = \frac{c}{a}\, e^{iwt}$$

Where '$c$' is the velocity of the light wave and '$a$' is the slit width which is comparable with the wavelength of the light.

Now let us consider the line element $dz$ is situated at $(0,z)$. The amplitude at any point P on the screen due to the line element $dz$ is given by




$$dy_p = \frac{c/a}{r} e^{i(wt-kr)} dz$$

Here, $r = r_0 - z\sin\theta$

Now as the value of $z\sin\theta$ is very small, we can ignore it in the denominator part but we can't ignore it in the exponential term.

Therefore,

$$dy_p = \frac{c/a}{r_0} e^{i(wt-kr_0)} e^{ikz\sin\theta} dz$$

$$= Re^{i\beta z} dz$$

Where $R = \frac{c/a}{r_0} e^{i(wt-kr_0)}$ and $\beta = k\sin\theta = \frac{2\Pi}{\lambda}\sin\theta$

Therefore the amplitude of the light wave at any point P on the screen due to the entire slit is given by

$$y_p = \int_{-a/2}^{+a/2} Re^{i\beta z} dz$$

$$= \frac{R}{i\beta} \left(e^{\frac{i\beta a}{2}} - e^{-\frac{i\beta a}{2}}\right)$$

$$= \frac{R}{i\beta} 2i \sin\left(\frac{\beta a}{2}\right)$$

$$= \frac{\frac{c/a}{r_0} e^{i(wt-kr_0)}}{\beta} 2 \sin\left(\frac{\beta a}{2}\right)$$

$$= \frac{c}{r_0} e^{i(wt-kr_0)} \frac{\sin\left(\frac{\beta a}{2}\right)}{\frac{\beta a}{2}}$$

Therefore, $y_p{}^* = \frac{c}{r_0} e^{-i(wt-kr_0)} \frac{\sin\left(\frac{\beta a}{2}\right)}{\frac{\beta a}{2}}$

Therefore the intensity at any point P due to the entire slit is given by

$$I_p = y_p y_p{}^*$$

$$= \left(\frac{c}{r_0}\right)^2 \frac{\sin^2\left(\frac{\beta a}{2}\right)}{\left(\frac{\beta a}{2}\right)^2}$$

$$= I_0 \left(\frac{\sin\alpha}{\alpha}\right)^2$$

Where, $I_0 = \left(\frac{c}{r_0}\right)^2$ and $\alpha = \left(\frac{\beta a}{2}\right)$

***<u>Condition for maxima and minima for single slit diffraction:</u>***

***<u>For minima:</u>***

$$\frac{\sin\alpha}{\alpha} = 0 \qquad \textit{but } \alpha \textit{ not equal to } 0.$$

$or,$ $\sin\alpha = 0 = \sin m\Pi$

$[where\ m = 1,2,3, \ldots .\ etc.\ but\ m\ is\ never\ equal\ to\ 0.]$

Therefore, $\alpha = m\Pi$

$$Or, \left(\frac{\beta a}{2}\right) = m\Pi$$

$$Or, \frac{a}{2}\frac{2\Pi}{\lambda}\sin\theta = m\Pi$$

Therefore, $a\sin\theta = m\lambda$

*This is the condition for minima.*

***For maxima:***

For maxima, $$\frac{dI_p}{d\alpha} = 0$$

$$Or, \quad \frac{d}{d\alpha}\left[I_0\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

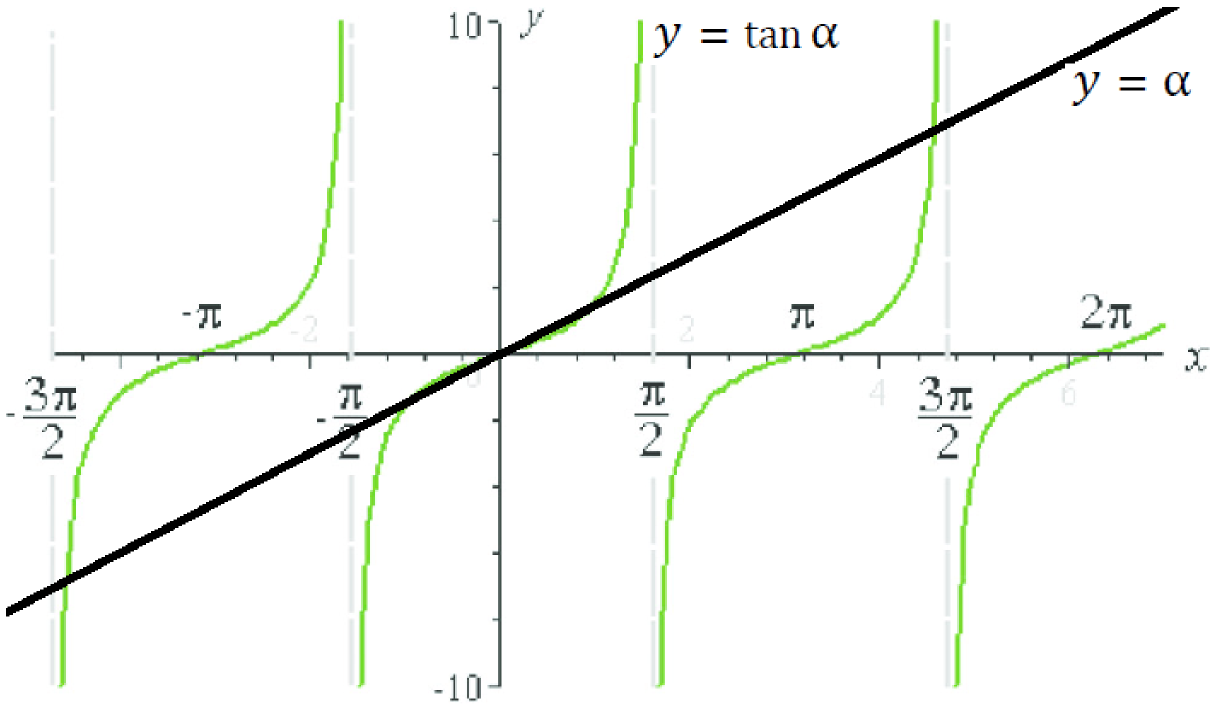
$$Or, \quad 2I_0\left(\frac{\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$Or, \quad \alpha\cos\alpha - \sin\alpha = 0$$

$$Or, \quad \tan\alpha = \alpha = y\ (say)$$

Therefore, $y = \alpha$ and $y = \tan\alpha$

Now this can be solved graphically.




Therefore, $\alpha = 0, \pm\frac{3\Pi}{2}, \pm\frac{5\Pi}{2}, \pm\frac{7\Pi}{2}, \pm\frac{9\Pi}{2}, \ldots\ldots etc.$

*This is the condition for maximum.*

## ***Intensity of the central-maxima, 1st, 2nd and other maxima:***

Intensity of the central maxima, when $\alpha = 0$

$$I_p = I_0 = \left(\frac{c}{r_0}\right)^2$$

Intensity of the 1st maxima, when $\alpha = \pm\frac{3\Pi}{2}$

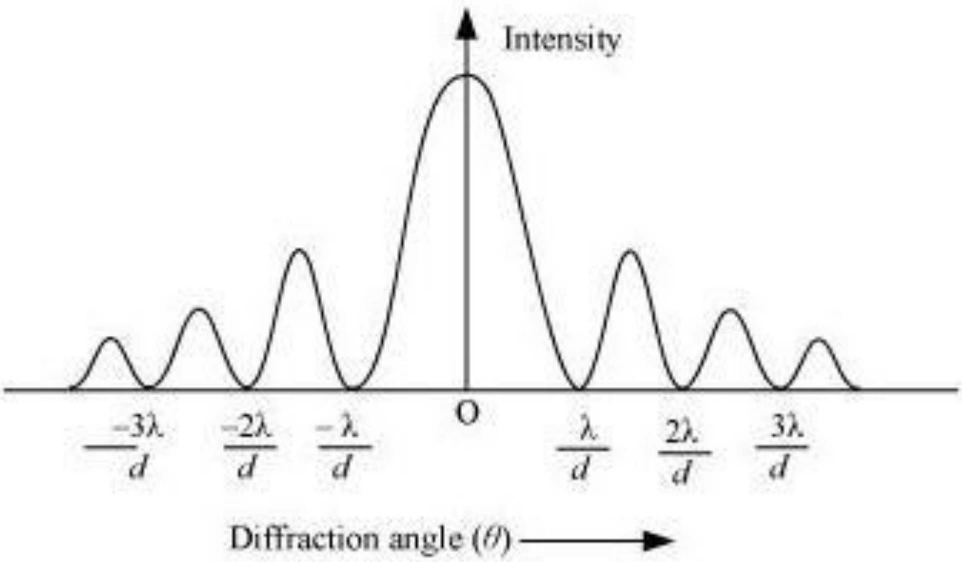
$$I_p = I_1 = \left(\frac{c}{r_0}\right)^2 \left(\frac{\sin\left(\pm\frac{3\Pi}{2}\right)}{\left(\pm\frac{3\Pi}{2}\right)}\right)^2 = \frac{I_0}{22}$$

Intensity of the 2nd maxima, when $\alpha = \pm\frac{5\Pi}{2}$

$$I_p = I_2 = \left(\frac{c}{r_0}\right)^2 \left(\frac{\sin\left(\pm\frac{5\Pi}{2}\right)}{\left(\pm\frac{5\Pi}{2}\right)}\right)^2 = \frac{I_0}{61}$$

Therefore, $$I_0 : I_1 : I_2 = 1 : \frac{1}{22} : \frac{1}{61}$$

## ***Graphical Representation of IDF for Single Slit Diffraction:***



## ***Home Work:***

1) What happen when slit width increases or decreases?
2) What happen when wave length of light increases or decreases?



***Please Stay Home & be Safe***