West Bengal State University

B.A./B.Sc./B.Com (Honours, Major, General) Examinations, 2015

PART – III

PHYSICS — HONOURS

Paper – V

Duration : 4 Hours] [Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

1. Answer any *ten* questions : $10 \times 2 = 20$

i) Starting from the Lagrangian of a free particle $\left(L = \frac{1}{2}m\dot{x}^2\right)$, obtain the Hamiltonian and set up the Hamilton's equations of motion.

ii) What is meant by non-holonomic constraint ? Give an example.

iii) What is meant by virtual displacement ? In what sense is it different from actual displacement ?

iv) For a particle with rest mass m_0 and velocity v prove that $E^2 - p^2c^2 = m_0^2c^4$, where the symbols have their usual meaning.

v) Does the density of an object change due to relative motion of the observer ? If yes, by what factor ?

vi) State the postulate of equal apriori probability.

vii) A system with allowed energy levels O and E is in equilibrium with a thermal heat bath at temperature T. If the probability of occupying the excited state is half that of the ground state, show that $T = E/(k_B \ln 2)$, where k_B is the Boltzman's constant.

viii) What is Fermi energy ? Explain briefly.

ix) A particle is represented by wave function $\psi(x) = Ae^{ip.x}$, where A is a constant. Is it consistent with uncertainty principle ?

x) Show that energy eigenstates of a particle moving in a one dimensional potential $V(x)$ have definite parity if $V(x) = V(-x)$.

xi) What is the Compton wavelength of a particle ? What is its significance ?

xii) Given $[x, p] = i\hbar$, find $[x, p^3]$.

xiii) Why operators associated with any dynamical variables are taken to be Hermitian ?

xiv) A normalized wave function $\psi(x,t) = \sum C_n(t)\psi_n(x)$, where ψ_n is a set of complete orthonormal eigenfunctions. Show that $\sum_n |C_n|^2 = 1$.

xv) Calculate the Lande-g factor for the atomic state $^2P_{1/2}$.

xvi) What is Raman effect ?

Group – A

(Answer any *one* question.)

2. a) State D'Alembert's principle. A frictionless block of mass m is placed on an incline making an angle α with the horizontal. The incline is now given a horizontal acceleration in the vertical plane of the incline such that the block cannot slide. Use D'Alembert's principle to find the acceleration given.

b) The point of suspension of a simple pendulum (length l, mass m) is moving on a horizontal line according to the relation $x = a\cos\omega t$ (a and ω are constants). Find the Lagrangian and show that the equation of motion for small angular displacement θ is given by $\ddot{\theta} + \frac{g}{l}\theta = \frac{a\omega^2}{l}\cos\omega t$. (1 + 4) + (3 + 2)

3. a) Find the positions and stability of the equilibrium for the given potential $V(x) = ax^2 + bx^3$ with a, b being constants. Draw the profile of the potential indicating your results.

b) Show that a dynamical variable $F = F(q, p, t)$ is a constant of motion if $\frac{\partial F}{\partial t} + [F, H]_{PB} = 0$, where H is the Hamiltonian of the system and PB denotes Poisson's bracket.

c) What is the criteria that a transformation $(q, p) \to (Q, P)$ is canonical ? Show that the transformation $P = aq + bp$; $Q = cq + dp$ is canonical if $ad - bc = 1$. ($a, b, c, d \to$ constants). 4 + 3 + 3

Group – B

(Answer any *one* question.)

4. a) Show that two successive Lorentz transformations in the same direction with velocity parameters β_1 and β_2 is equivalent to a single Lorentz transformation with velocity parameter given by $\beta = (\beta_1 + \beta_2)/(1 + \beta_1\beta_2)$. 4

b) A particle is moving in one dimension. Draw the world line of the particle if it is at rest. What is the nature of the world line of a photon ? Show graphically. ($1\frac{1}{2} + 1\frac{1}{2}$)

c) Distinguish between space-like, time-like and light-like intervals. 3

5. a) Muons have life-time $\sim 2 \times 10^{-6}$ sec and speed 0·998 c with respect to ground observer. How much distance it may travel before decaying with respect to ground observer. Derive the relation that you use. 2 + 3

b) Define what is meant by proper velocity. 2

c) A particle has relativistic energy-momenta $(E, \vec{p})$. Measured from another frame it is $(E', \vec{p}')$, where the 2nd frame moves with speed v along the direction of the particle with respect to the first frame. Find the relation between $(E', \vec{p}')$ and $(E, \vec{p})$. 3

Group – C

(Answer any *two* questions.)

6. a) Given $\Gamma(k+1) = k! = \int_0^\infty dx\, x^k e^{-x}$. Show that $f(x) = x^k e^{-x}$ is a sharply peaked function with peak at $x = k$ and the integral can be approximated by $k! \approx k^k e^{-k} \sigma_k$, where σ_x is the width of the peak. Assuming σ_k too small compared to k, show that $ln\, k! = k\, ln\, k - k$. 3

b) N coins are lined up in a straight chain. A macrostate (N, H) is defined such that there are total H number of heads while rest of the coins being tails. Find total number of

i) all possible macrostates of the system,

ii) all possible microstates (X) of the system,

iii) all allowed microstates Ω (N, H) corresponding to the macrostate (N, H),

Using Stirling formula, show that most probable macrostate is obtained for $H = N/2$,

Show that under this approximation $\Omega\left(N, \frac{N}{2}\right) \approx X$.

1 + 1 + 1 + 2 + 2

7. a) Distinguish between micro-canonical and canonical ensemble.

b) For a system represented by a canonical ensemble at temperature T, probability P_r of a microstate having energy E_r is given by $P_r = \frac{1}{Z} e^{-\beta E_r / kT}$, where symbols have their usual meaning.

Show that (i) internal energy $U \equiv \overline{E}_r = -\frac{\partial}{\partial \beta} \ln Z$, (ii) Helmholtz free energy, $A = -\beta \ln Z$, and (iii) entropy, $S = -k \sum_r P_r \ln P_r$.

c) Assuming the formula for entropy above is valid for microcanonical ensemble, show that $S = k \ln \Omega (E_r)$, where $\Omega (E_r)$ is the total number of microstates with energy E_r.

2 + (2 + 2 + 2) + 2

8. a) Derive Fermi-Dirac distribution function $f(E)$, stating clearly the assumptions made in the derivation.

b) Define density function g (E). Find an expression for g (E) in the case of a two-dimensional free electron gas contained in an area A.

c) Show that the number of electrons per unit area is given by

$$n \equiv \frac{N}{A} = \frac{1}{A} \cdot \int_0^\infty \mathrm{d}E \; g(E) \; f(E) = \frac{4\pi m_e k_B T}{h^2} \ln \left(e^{E_F / k_B T} + 1 \right)$$

[All symbols have usual meaning.]

4 + 3 + 3

Group – D

(Answer any *three* questions.)

9. a) Using the general definition of the expectation value of an operator, prove that $\frac{d}{dt} < x > = \frac{1}{m} < p_x >$, where the symbols have their usual meanings.

b) The ground state wave function of a one-dimensional harmonic oscillator (of mass m and classical frequency ω) is

$$\psi_0(x) = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-\frac{\alpha^2 x^2}{2}}$$

where $\left(\alpha = \sqrt{\frac{m\omega}{\hbar}}\right)$.

What is the energy corresponding to this state ? Discuss with proper explanation, whether this state is an eigenfunction of the momentum operator or not.

c) What is the physical meaning of the expectation value of an observable ? From physical correlation, would you expect a non-zero value $< \vec{p} >$ for a bound state ? 3 + (1 + 3) + (1 + 2)

10. a) To qualify as wavefunction, a function must be continuous and square integrable, justify on what physical ground these two criteria are imposed defining clearly what is meant by square integrability of a function.

b) Write down the time-independent Schrödinger equation in one-dimension for the potential $V(x)$.

i) Show that the first derivative of the wavefunction $\left(\frac{\partial\psi}{\partial x}\right)$ is continuous for all x if $V(x)$ is continuous or at most has finite No. of finite discontinuity.

ii) What should be the nature of potential $V(x)$ such that $\left(\frac{\partial\psi}{\partial x}\right)$ is not continuous at a point x_0.

iii) What are the physical implications of the above results ? (1 + 2) + (2 + 2 + 3)

11. a) Determine the probability current density for the wave-packet represented by $\psi(x) = e^{-\alpha^2 x^2/2} e^{ikx}$.

b) The wave-function of a free particle confined in a box is given by $\psi_n(x) = \sqrt{\frac{2}{L}} . \sin\left(\frac{n\pi x}{L}\right)$. Show that the probability to find the particle in any small interval (Δx) lying between x and $x + \Delta x$ is independent of x if n is large.

c) Show that the operator P is linear if $P\psi(x) = \psi(-x)$.

d) Find the expectation value of $\left(L_x^2 + L_y^2\right)$ in a state having $l = 1$ and $m = -1$, symbols have their usual meaning. 3 + 3 + 2 + 2

12. a) For a one-dimensional harmonic oscillator with mass m and angular frequency ω, define $\hat{a}_{\mp} = \sqrt{\frac{m\omega}{2h}}\left(\hat{x} \pm i\frac{\hat{p}x}{m\omega}\right)$. Using the basic commutation relation between $\hat{x}$ and $\hat{p}x$, show that (i) $\left[\hat{a}_-, \hat{a}_+\right] = 1$ and $\hat{H} = \hbar\omega\left(\hat{a}_+ \hat{a}_- + \frac{1}{2}\right)$, where $\hat{H}$ is the Hamiltonian of the system.

b) If ψ_n and λ_n are the eigenstates and eigenvalues of the operator $\hat{N} \equiv \hat{a}_+ \hat{a}_-$, show that $\left(\hat{a}_\pm \psi_n\right)$ are also eigenstates of $\hat{N}$ with eigenvalue $(\lambda_n \pm 1)$. Hence find the eivenvalue of $\hat{H}$.

c) Azimuthal part of the wave-function of a particle in a central potential is given by $\Phi(\varphi) = A_e{}^{im\phi}$, where A and m are constants. Show that m should be an integer. 4 + 4 + 2

Group – E

(Answer any *one* question.)

13. a) i) What is meant by space-quantization ? What role does magnetic quantum number play in space-quantization ? Explain in the light of vector atom model.

ii) "Uncertainty principle prohibits the angular momentum vector $\vec{L}$ from having a definite direction." — Explain why. Check if this fact is incorporated into the vector atom model.

b) In Stern-Garlach experiment, what is the expected result if atomic dipoles are randomly oriented ? What was the observed result ? How the result is explained ? (3 + 3) + (1 + 1 + 2)

14. a) i) Explain what is meant by spin-orbit coupling.

ii) Show how spin-orbit coupling explains the fine structure splitting of $3P \rightarrow 3S$ transition in sodium to D_1 and D_2 lines.

b) i) State Hund's rules to determine the ground state of a many-electron atom using *L-S* coupling scheme.

ii) Show that the ground state of nitrogen $(1s^2 2s^2 2p^3)$ is given by $^4S_{3/2}$ using Hund's rule. (2 + 3) + (3 + 2)