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West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2015

PART-III

MATHEMATICS– Honours

Paper– VIII (A)

Duration : 2 Hours | Full Marks : 50

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

(Notations used have their usual meanings.)

Group-A

Section - I

(*Linear Algebra*)

Answer any *one* question from the following : $1 \times 10 = 10$

1. a) If $T : IR^3 \to IR^2$ is given by

$$T(a_1, a_2, a_3) = (a_1 + a_3, a_1 + a_2 + 2a_3)$$

then find Ker T. 2

b) If V and W are finite dimensional vector spaces over a field F then show that V and W are isomorphic if and only if $\dim V = \dim W$. 4

c) Let V be a vector space of all real polynomials of degree ≤ 3 and $T : V \to V$ be defined by $T(p(x)) = x\frac{d}{dx}(p(x))$.

Show that T is a linear transformation. Find the matrix of T with respect to the ordered basis $\left\{ 1, x, x^2, x^3 \right\}$. 4

2. a) Let M_{mn} be the collection of all $m \times n$ matrices over IR and $T : M_{mn} \to M_{nm}$ be given by $T(A) = A^T$.

Is T a linear transformation ? 2

b) Let V be the vector space of all real polynomials of degree ≤ 2 and $T : V \to V$ be given by

$$T\,(at^2+bt+c)=(a+2b)\;t+(b+c).$$

(i) Is $-4t^2+2t-2$ in Ker T ?

(ii) Is t^2+2t+1 in range of T ?

(iii) Find a basis of the null space of T.

(iv) Find the rank of T. 2 + 2 + 2 + 2

Section - II

(Modern Algebra)

Answer any *one* question from the following : $1 \times 8 = 8$

3. a) Let G be a group and H a normal subgroup of G. If G/H is cyclic then show that G is Abelian. 3

b) If index of a subgroup H in a group G is 2 then show that H is normal in G. 2

c) Consider the group $GL\,(2, \mathbb{R})$ of all 2×2 non-singular matrices under matrix multiplication and the group $\mathbb{R}^*$ of all non-zero real numbers under usual multiplication. Define $f : GL\,(2, \mathbb{R}) \to \mathbb{R}^*$ by $f(A)=\det A$, where det A denotes the determinant value of A. Show that f is a group homomorphism. Find Ker f. Is the quotient group $GL\,(2, \mathbb{R})/\text{Ker } f$ cyclic ? 3

4. a) In a group G, if H and K are subgroups such that K is normal in G, prove that $KH=\{\,kh \;:\; k\in K \text{ and } h\in H\,\}$ is a subgroup of G. 3

b) Either prove or disprove : $(\mathbb{R}, +)$ and $(\mathbb{R}^+, \cdot)$ are isomorphic. ($\mathbb{R}^+$ denotes the set of all positive real nos.) 2

c) If H is a subgroup of G then show that the subgroup $\bigcap_{g\in G} gHg^{-1}$ is normal in G. 3

Section- III

(Boolean Algebra)

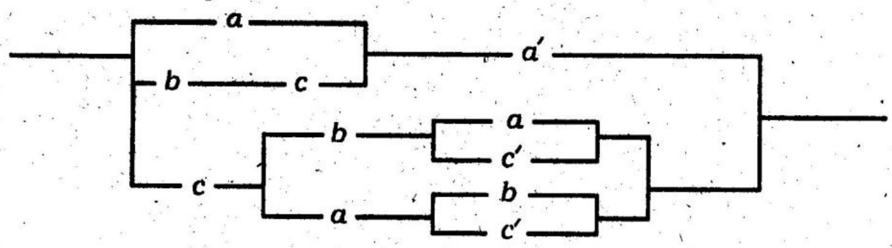
Answer any *one* question from the following : 1 × 7 = 7

5. a) In a Boolean Algebra ($(B, +, \bullet, ')$), show that

(i) $a + a = a \quad \forall a \in B$

(ii) $(a+b)' = a' . b' \quad \forall a, b \in B$ 1 + 3

b) Find the Boolean function that represents the circuit below and hence find a simpler circuit. 1 + 1 + 1



6. a) Let $f = xyz + x'yz + xy'z + xyz'$. Convert the given Boolean function in *dnf* into its conjunctive normal form (*cnf*). 3

b) A committee consisting of three members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seat. Design as simple a circuit as possible which will allow current to pass when and only when a proposal is approved. 4

Group - B

(Differential Equations - III)

Answer any *one* question from the following : 1 × 15 = 15

7. a) Obtain the series solution of $\frac{d^2y}{dx^2} - 4y = 0$, satisfying $y(0) = 1$ and $y'(0) = 1$. 5

b) Find the Laplace transform of $a + bt + \frac{c}{\sqrt{t}}$. 5

c) Solve the boundary-value problem $(D^2 + 9)\, y = \cos 2t$, given that $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = (-1)$ 5

8. a) Applying power series method, solve

$$\frac{d^2y}{dx^2} - y = x$$ 5

b) Compute $L^{-1}\left\{\frac{3}{s^4(s^2+1)}\right\}$ by using convolution theorem. 5

c) Solve the equation $y'' + 2y' + y = 0$ given that $y(0) = 0$ and $y(1) = 2$. 5

Group - C

(*Tensor Calculus*)

Answer any *one* question from the following : 1 × 10 = 10

9. a) Define the null vector. The Line element in the V_4 space is given by $ds^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$. Verify whether $(-1, 0, 0, \frac{1}{c})$ is the null vector in V_4. 1 + 3

b) If $\delta_{ij} = \begin{cases} 1, & \text{when } i = j \\ 0, & \text{when } i \neq j \end{cases}$

then prove or disprove : δ_{ij} are the components of a covariant tensor of second order. 3

c) Prove that all Christoffel symbols are zero in the Euclidean space. 3

10. a) Define the covariant differentiation of tensors in the direction of a vector field X. 3

b) If A_i is a covariant vector, determine whether $\frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$ are the components of a tensor or not. 3

c) Prove that the covariant derivatives of the fundamental metric tensor and the Kronecker delta in a Riemannian space are zero. 2 + 2
