West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2015

PART-III

MATHEMATICS– Honours

Paper– VII

Duration : 4 Hours | Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Group-A

(VECTOR ANALYSIS-II)

[Marks : 10]

Answer any *one* question : $1 \times 10 = 10$

1. a) Find the constants a, b, c so that the vector

$$\vec{V} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$$

is irrotational and then assuming $\vec{V} = \vec{\nabla}\phi$ obtain ϕ. 5

b) Show that $\vec{F} = (2xy+z^2)\vec{i} + x^2\vec{j} + 3xz^2\vec{k}$ is a conservative force field. Find the scalar potential. Find also the work done in moving an object in this field from (1, –2, 1) to (3, 1, 4). 5

2. a) Verify Green's theorem in a plane for $\oint_C \left\{ (x^2+xy)\,dx + xdy \right\}$, where C is the curve enclosing the region bounded by $y = x^2$ and $y = x$. 5

b) Apply Stokes' theorem to evaluate $\int_\Gamma (\sin z dx - \cos x dy + \sin y dz)$, where Γ is the boundary of the rectangle $0 \le x \le \pi;\ 0 \le y \le 1,\ z = 3$. 5

Group-B

(ANALYTICAL STATICS)

(Marks : 35)

Answer any *five* questions : 5 × 7 = 35

3. Explain astatic equilibrium of a system of coplanar forces acting at different points of a body and obtain the astatic centre.

4. A thin hemispherical bowl of radius b and weight W rests in equilibrium on the highest point of a fixed sphere of radius a which is rough enough to prevent any sliding. Inside the bowl is placed a small smooth sphere of weight w. Show that the equilibrium is not stable unless $w < W.\frac{a-b}{2b}$.

5. State and prove the principle of virtual work for a system of coplanar forces acting on a rigid body.

6. Let X, Y, Z and L, M, N denote respectively the algebraic sum of the components of a system of forces and their moments with respect to Cartesian axes Ox, Oy, Oz passing through any base point O. Show that $X^2+Y^2+Z^2$ and $LX+MY+NZ$ are invariant whatever be the base point of the direction of the axes.

7. Two forces act, one along the line $y = 0$, $z = 0$ and other along the line $x = 0$, $z = c$. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2 + y^2)\ z = cy^2$.

8. A solid homogeneous hemisphere rests on a rough horizontal plane and against a rough vertical wall, the coefficients of friction being μ and μ' respectively. Show that the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8\mu}{3}\ \frac{1+\mu'}{1+\mu\mu'}\right)$.

9. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that

$$\tan\phi = \frac{3}{8} + \tan\theta.$$

10. Find the centre of gravity of the surface generated by the revolution of a loop of the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line.

11. Forces X, Y, Z act along three straight lines $y = b$, $z = -c$; $z = c$, $x = -a$; $x = a$, $y = -b$; respectively. Show that they will have a single resultant if $\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$.

Group - C

(RIGID DYNAMICS)

[Marks : 30]

Answer any *two* questions : $2 \times 15 = 30$

12. a) Show that the momental ellipsoid at the centre of an ellipsoid is

$$(b^2+c^2)x^2+(c^2+a^2)y^2+(a^2+b^2)z^2 = \text{constant}.$$ 8

b) A plank of mass M is initially at rest along a straight line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M'a}{(M+M')g\sin\alpha}}$ where a is the length of the plank and g is the acceleration due to gravity . 7

13. a) Explain the two-dimensional motion of a rigid body. Show that the kinetic energy of a rigid body moving on two dimensions is given by $\frac{1}{2}MV^2+\frac{1}{2}MK^2\dot{\theta}^2$ in usual notations to be explained by you. 8

b) An elliptic lamina is rotating in its own plane about one of its foci with angular velocity ω. This focus is set free and the other focus is fixed at the same instant. Show that the resulting angular velocity is $\frac{\omega}{3}$, if the eccentricity of the ellipse is $\frac{\sqrt{2}}{3}$. 7

14. a) Define 'Impulse of a force'. Obtain the equations of motion in two dimensions of a rigid body moving under impulsive forces. 7

b) A compound pendulum of mass M oscillating about a fixed horizontal axis has its centre of oscillation at C. Find the period of oscillation of the compound pendulum. Show further that the period is unaltered even if a weight is rigidly attached to the body of the pendulum at C. 8

Group - D

(HYDROSTATICS)

[Marks : 25]

Answer any one question from each Section.

Section - I

15. a) A given mass of liquid is in equilibrium under the action of a system of conservative forces. Write down the pressure equation at a point. Show that the surfaces of equi-pressure, equi-density coincide. If the system of forces is the gravity only, show that these surfaces are horizontal. 8

b) If the floating solid be a cylinder, with its axis vertical, the ratio of whose specific gravity to that of the fluid is σ, prove that the equilibrium will be stable if the ratio of the radius of the base to the height be greater than $\left[\, 2\sigma\,(1-\sigma)\,\right]^{1/2}$. 7

16. a) Prove that a cylinder of radius a and length $\frac{a}{n}$ cannot float upright in stable equilibrium if its specific gravity lies between

$$\frac{1}{2}\left[1-\sqrt{1-2n^2}\right] \text{ and } \frac{1}{2}\left[1+\sqrt{1-2n^2}\right]$$ 8

b) A lamina in the shape of a quadrilateral $ABCD$ has its side CD in the surface of a liquid and the sides AD, BC vertical and equal to α, β respectively. Show that the depth of the centre of pressure is $\frac{1}{2}\frac{(\alpha^2+\beta^2)(\alpha+\beta)}{(\alpha^2+\beta^2+\alpha\beta)}$. 7

Section - II

17. a) Two fluids of different densities at rest under gravity do not mix. Prove that their surface of separation is a horizontal plane. 5

b) A cone of density ρ whose height is h and the radius of whose base is a is floating with its axis vertical and vertex upward in liquid of density σ. Prove that the equilibrium is stable if $\rho < \sigma(1-\cos^6\alpha)$. 5

18. a) If the absolute temperature T at a height z be a given function $f(z)$ of the height, then show that the ratio of the pressures at two heights z_1 and z_2 is

$$\log\frac{p_2}{p_1} = -\frac{g}{R}\int_{z_1}^{z_2}\frac{dz}{f(z)}$$; R being the constant in the equation $p = R\rho T$. 5

b) A liquid fills the lower half of a circular tube of radius a in a vertical plane. If the tube is now rotated about the vertical diameter with uniform angular velocity ω such that the liquid is about to separate in two parts, show that $\omega = \sqrt{\frac{2g}{a}}$. 5
