West Bengal State University

B.A./B.Sc./B.Com. (Honours, Major, General) Examinations, 2015

PART-III

MATHEMATICS– Honours

Paper– VI

Duration : 4 Hours Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Group-A

Answer any *two* questions from Q. Nos. **1** to **3** and any *one* from Q.Nos. **4** and **5**.

1. Answer any *three* of the following questions : $3 \times 5 = 15$

a) State the axioms of probability. Define certain and impossible events. Give examples for both of them. 2 + 3

b) For two arbitrary events A and B defined on the event space S, show that $P(B/A) \geq 1 - \frac{P(\overline{B})}{P(A)}$, $P(A) \neq 0$. 5

c) State and prove Bayes' theorem. 5

d) If $6n$ tickets numbered $0, 1, 2, \ldots\ldots, 6n-1$ are placed in a bag and three are drawn out, show that the chance that the sum of the numbers on them is equal to $6n$ is $\frac{3n}{(6n-1)(6n-2)}$. 5

e) The chances of three students A, B and C for solving a problem are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability of the problem being solved ? 5

[Turn over

2. Answer any *three* of the following : $3 \times 5 = 15$

a) If X is a standard normal variate, find the mean and variance of e^X. 5

b) Show that the first absolute moment about the mean for the normal (m, σ) distribution is $\sqrt{\frac{2}{\pi}}\ \sigma$. 5

c) Define Poisson distribution. Prove that the sum of two independent Poisson variates with parameters μ_1 and μ_2 is again a Poisson variable with parameter $\mu_1 + \mu_2$. 5

d) Show that if X is a binomial variate then $\operatorname{cov}\left(\frac{X}{n}, \frac{n-X}{n}\right) = -\frac{pq}{n}$. 5

e) Show that in a Poisson distribution with unit mean, mean deviation about mean is $\frac{2}{e}$ times the standard deviation. 5

3. Answer any *three* of the following : $3 \times 5 = 15$

a) Find k such that $\rho_{uv} = 0$, where $U = X + kY$ and $V = X + \frac{\sigma_x}{\sigma_y} Y$. 5

b) Two discrete random variables X and Y have the joint probability density function

$$f(x, y) = \frac{\lambda^x e^{-\lambda} p^y (1-p)^{x-y}}{y!\,(x-y)!}, \quad x = 0, 1, 2 \ldots;\ y = 0, 1, 2 \ldots$$

where λ, p are constants with $\lambda > 0$ and $0 < p < 1$

(i) Find the marginal *pdfs* of X and Y.

(ii) Find the conditional distribution of Y for a given X. 4 + 1

c) If X be a normal (m, σ) variate then prove that

$\mu_{2r+2} = \sigma^2 \mu_{2r} + \sigma^3 \frac{d\mu_{2r}}{d\sigma}$. Use this result to find the coefficient of Kurtosis β of this distribution. 5

d) Let X_i assumes the values i and $-i$ with equal probability. Show that the law of large numbers cannot be applied to the variables $X_1, X_2, \ldots X_n$. 5

e) Show that standard Gamma variate tends to standard normal variate as $n \longrightarrow \infty$. (n is the parameter of the Gamma distribution) 5

4. a) A population is defined by the density function

$$f\ (x,\ \alpha) = \frac{x^{L-1} e^{-x/\alpha}}{\Gamma\ (l)\ \alpha^l},\ 0 < x < \infty,\ l \text{ being constant.}$$

Estimate the parameter α by the method of maximum likelihood and show that the estimate is consistent and unbiased. 7

b) If a linear relation $aX + bY + c = 0$ exists between the variables X and Y and $ab < 0$, then find the coefficient of correlation between X and Y. 5

c) Prove that, for a simple random sample, the sample variance s^2 is an unbiased estimate of the population variance S^2. 5

d) If one of the regression coefficients is more than unity, prove that the other must have been less than unity. 3

5. a) The regression lines for a bivariate sample are given by $x+2y-5=0$ and $2x+3y-8=0$ and let $S_x^{\,2}=12$. Calculate the values of $\bar{x}$, $\bar{y}$, S_y and r. 5

b) Given $f(x, \theta)=\begin{cases}(1+\theta)\, x^{\theta}, & 0 \le x \le 1 \\ 0, & \text{elsewhere}\end{cases}$

with $\theta > 0$. If the hypothesis $H_0 : \theta = 1$ is to be tested by taking a single observation on x-axis and using the interval $x < 0{\cdot}5$ as the critical region then

(i) Calculate the type I error

(ii) Calculate the probability of determining that H_0 is false if the true value of θ is 2. 5

c) Use Neyman-Pearson theorem to construct a test of null hypothesis $H_0 : \mu = \mu_0$ against an alternative $H_1 : \mu = \mu_1$ for a normal (μ, σ) population, where σ is known with $\mu_0 \neq \mu_1$. 6

d) The mean of 10 readings on the length of a given rod is 20 inches. The standard deviation of errors of measurement is known to be $0{\cdot}1$ inch. Does the result contradict the assumption that the length of rod is $19{\cdot}9$ inches ? 4

Group - B

Answer any *three* questions from Section - I and any *two* from Section - II.

Section - I

[Marks : 30]

6. a) Show that

(i) $(1+\Delta)(1-\nabla)=1$

(ii) $\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$, where the interval of differencing is h. 2 + 3

b) Discuss the propagation of error in a difference table and deduce the relation $\Delta^k \varepsilon_j = \sum_{i=0}^{k} (-1)^i \binom{k}{i} \varepsilon_{j+k-i}$, the symbols having usual meaning.

4 + 1

7. a) Deduce Newton's forward interpolation formula with error term. 7

b) Obtain a numerical differentiation formula from the above formula for 2nd order differentiation. 3

8. a) Find the Simpson's $\frac{1}{3}$ composite rule for numerical integration and discuss the error term. 5

b) Describe the Gauss-Seidel method to solve numerically a system of n linear equations with n variables. 5

9. a) Show that, in approximating $f(x)$ by the interpolation polynomial using distinct points $x_0, x_1, x_2, \ldots x_n$, the remainder is of the form

$$(x-x_0)(x-x_1)\ldots(x-x_n)\frac{f^{(n+1)}(\xi)}{(n+1)!}$$

where min $\{x, x_0, \ldots x_n\} < \xi <$ max $\{x, x_0, \ldots, x_n\}$ 5

b) Explain the Newton-Raphson method for computing a simple real root of $f(x)=0$. Can we apply this method to the equation $x^2 - x + \frac{1}{4} = 0$? Justify your answer. 4 + 1

10. a) Describe the power method to compute the greatest eigenvalue of a real square matrix of order n. 5

b) Use Picard's method to solve $\frac{dy}{dx} = -xy$ up to he fourth approximation where $y(0) = 1$. 5

Section - II

[Marks : 20]

11. Answer any *five* of the following : $5 \times 2 = 10$

(i) What is an input device of a computer ? Give examples.

(ii) What do you mean by an ASCII code ?

(iii) Which of the following is/are not an output device ?

Monitor, Sound box, Printer, Web cam.

Give reasons.

(iv) What is a flowchart ? Draw a flowchart for adding three numbers a, b and c.

(v) Write the full forms of CPU, ALU, HLL and CNF.

(vi) Use 2's complement method to obtain $(110110)_2 - (10110)_2$.

(vii) Find the hexadecimal equivalent of $(41819)_{10}$.

(viii) What is meant by read-only memory ? Is ROM a random access memory ? Justify.

12. a) Write a FORTRAN or C program to arrange the following set of real numbers in descending order :

7·1, 7·2, 8·1, 8·9, 3·2, 1·6, 7·5. 5

b) Write a program in FORTRAN or C to evaluate the integral

$\int_0^{1\cdot2} e^{-8x^2} \cos x \, dx$ by Trapezoidal rule, taking $n = 12$. 5

13. a) Explain the uses of if-else and do-while statements in C with suitable examples. 5

b) Write a C or FORTRAN program to determine whether a number is prime or not. 5
